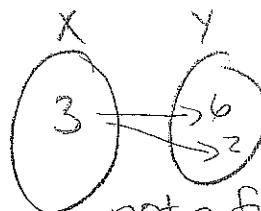
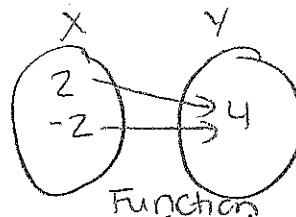
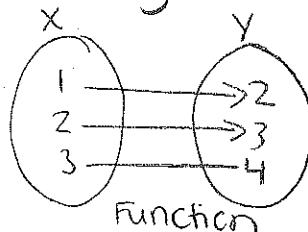


Midterm Review: Function Foundations

1. (a) What is a function? Give examples of mappings.

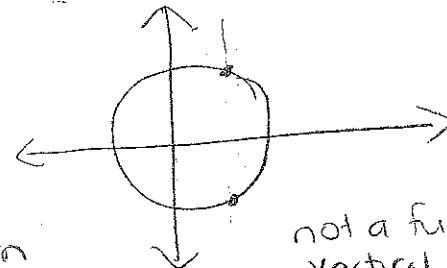
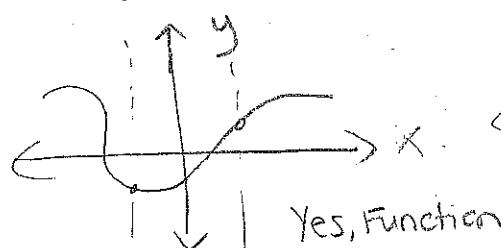
Every input has exactly one output.



not a function

- (b) If given a graph, what test can you use to determine whether it is a function or not? Give examples.

Vertical line test.



*not a function.
vertical line creates 2 points.*

- (c) What is domain? *X-values
inputs*

- (d) What is range? *y-values
outputs*

2. (a) Write the following domain in inequality notation: $[2, \infty)$

$$x \geq 2$$

- (b) Write the following domain in interval notation: $-5 \leq x < 6$

$$[-5, 6)$$

- (c) Write the following range in inequality notation: $(-\infty, -3)$

$$y < -3$$

- (d) Write the following range in interval notation: $y \geq -4$

$$[-4, \infty)$$

3. Given the relation $(2, 4), (-1, 3), (-2, 4)$, and $(6, 9)$

- (a) Is it a function?

Yes

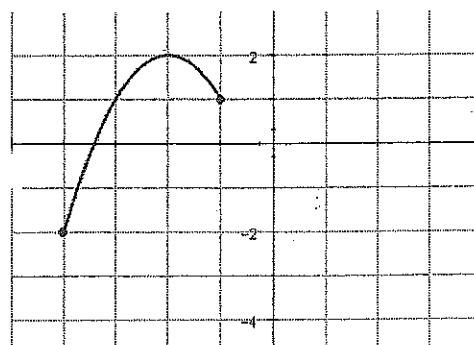
- (b) What is the domain?

$$\{-2, -1, 2, 6\}$$

- (c) What is the range?

$$\{3, 4, 9\}$$

- 4.

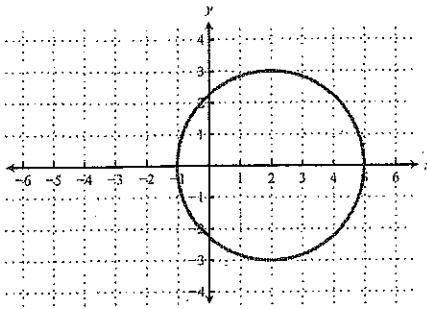


- (a) Is it a function? *Yes*

- (b) What is the domain? $[-4, -1]; -4 \leq x \leq -1$

- (c) What is the range? $[-2, 2]; -2 \leq y \leq 2$

5.



(a) Is it a function? NO

(b) What is the domain? $[-1, 5]$; $-1 \leq x \leq 5$ (c) What is the range? $[-3, 3]$; $-3 \leq y \leq 3$

6. Use the functions below to answer each question.

$$b(x) = x^2 + 2$$

$$c(x) = 3$$

$$d(x) = 2x + 3$$

$$f(x) = x^3$$

Determine each of the following.

a. $(b \circ d)(x)$

$$(2x+3)^2 + 2$$

$$(2x+3)(2x+3) + 2$$

$$4x^2 + 6x + 6x + 9 + 2$$

$$\boxed{4x^2 + 12x + 11}$$

b. $(b \circ d)(x)$

$$(x^2+2)(2x+3)$$

$$\boxed{2x^3 + 3x^2 + 4x + 6}$$

c. $b(f(c(x)))$

$$b(f(3))$$

$$\boxed{b(3^3)}$$

d. $b(x)d(x)$

$$(x^2+2)(2x+3)$$

$$\boxed{2x^3 + 3x^2 + 4x + 6}$$

e. $b(f(x))$

$$b(x^3)$$

$$(x^3)^2 + 2$$

$$\boxed{x^6 + 2}$$

f. $(d \circ f)(x)$

$$d(x^3)$$

$$2(x^3) + 3$$

$$\boxed{2x^3 + 3}$$

g. $2b(x) - 3d(x)$

$$2(x^2+2) - 3(2x+3)$$

$$2x^2 + 4 - 6x - 9$$

h. $(d - c)(x)$

$$2x + 3 - 3$$

$$\boxed{2x}$$

i. $d(b(f(x)))$

$$d(b(x^3))$$

$$d((x^3)^2 + 2)$$

$$d(x^6 + 2)$$

$$2(x^6 + 2) + 3$$

$$2x^6 + 4 + 3$$

$$\boxed{2x^6 + 7}$$

j. $(f \circ c)(x)$

$$f(3)$$

$$(3)^3$$

$$\boxed{27}$$

k. $(d \circ b)(x)$

$$d(x^2+2)$$

$$2(x^2+2) + 3$$

$$2x^2 + 4 + 3$$

$$\boxed{2x^2 + 7}$$

l. $(b \circ f)(x)$

$$(x^2+2)(x^3)$$

$$\boxed{x^5 + 2x^3}$$

7. Given $f(x) = 2x^2 - 8$ and $g(x) = 3x + 8$, determine each of the following.

a. $f(-3)$

$$2(-3)^2 - 8$$

$$2(9) - 8$$

$$18 - 8$$

$$\boxed{10}$$

b. $3f(2)$

$$3(2(2)^2 - 8)$$

$$3(2(4) - 8)$$

$$3(8 - 8)$$

$$3(0)$$

$$\boxed{0}$$

c. $(f-g)(-1)$

$$f(-1) - g(-1)$$

$$[2(-1)^2 - 8] - [3(-1) + 8]$$

$$(2 - 8) - (-3 + 8)$$

$$-6 - (5)$$

$$\boxed{-11}$$

d. $g(x) = 14$

$$\begin{array}{r} 3x + 8 = 14 \\ -8 \quad -8 \\ \hline 3x = 6 \\ \hline 3 \quad 3 \end{array}$$

$$\boxed{x=2}$$

e. $f(x+2)$

$$2(x+2)^2 - 8$$

$$2[(x+2)(x+2)] - 8$$

$$2(x^2 + 2x + 2x + 4) - 8$$

$$2(x^2 + 4x + 4) - 8$$

$$2x^2 + 8x + 8 - 8$$

$$\boxed{2x^2 + 8x}$$

f. $f(4x)$

$$2(4x)^2 - 8$$

$$2(16x^2) - 8$$

$$\boxed{32x^2 - 8}$$

g. $2g(-x)$

$$2(3(-x) + 8)$$

$$2(-3x + 8)$$

$$\boxed{-6x + 16}$$

h. $3f(0)$

$$3[2(0)^2 - 8]$$

$$3(0 - 8)$$

$$3(-8)$$

$$\boxed{-24}$$

8. A function has domain $\{-3, -2, 1, 4\}$. Another function has domain $[-3, 4]$.

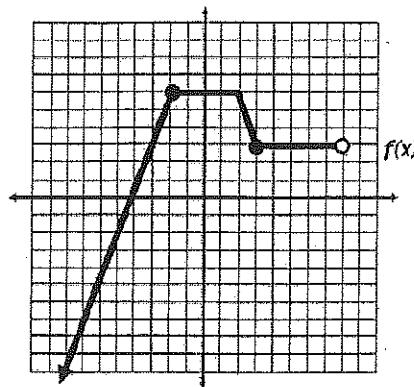
How are these domains different?

$\{-3, -2, 1, 4\}$ = Set notation. The x-values are only $-3, -2, 1$ and 4 . They are discrete points.

$[-3, 4]$ = interval notation. The x-values include -3 and 4 and every number between them.

Ex: $-7.5, -1.31, -2.6, \dots$

9. State the domain and range of each function in both interval notation and inequality notation.



Domain: $(-\infty, 8)$; $x < 8$

Range: $(-\infty, 6]$; $y \leq 6$

$f(x) = -5$, what is x ? $x = -6$

What is $3f(-4)$? $3(0)$

$$\boxed{0}$$

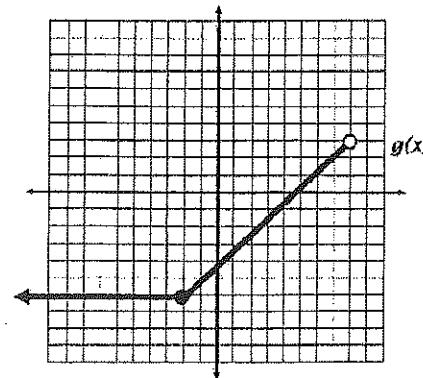
What is $f(0) - f(5)$?

$$6 - 3$$

$$\boxed{3}$$

$f(x) = 6$. What is x ?

$$x = [-2, 2]$$



Domain: $(-\infty, 8)$; $x < 8$

Range: $[-6, 3)$; $-6 \leq y < 3$

$g(x) = 0$, what is x ? $x = 5$

What is $2g(-10)$?

$$2(-6)$$

$$\boxed{-12}$$

What is $g(-15) + g(0)$?

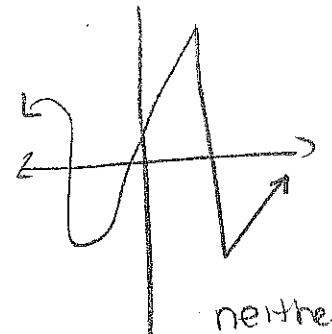
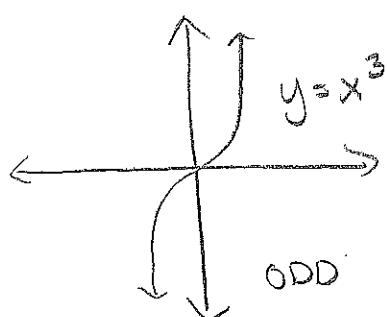
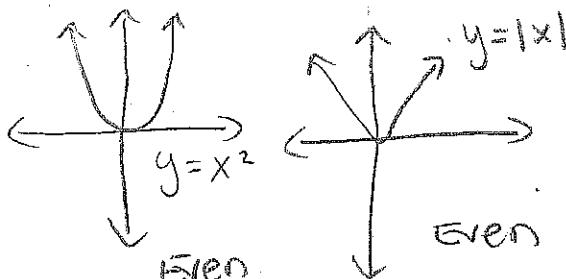
$$-6 + 4$$

$$\boxed{-10}$$

What is $g(10)$?

no solution

10. What does it mean for a function to be Even, Odd, or neither? Provide a visual example of each.
- Even: $f(-x) = f(x)$ "the function stays the same" Symmetrical about y -axis
- Odd: $f(-x) = -f(x)$ "all signs are opposite" Symmetrical about origin
- Neither: doesn't do either



11. Is the function even, odd, or neither? Show algebraically.

$$f(x) = -3x^2 + 4$$

$$f(-x) = -3(-x)^2 + 4$$

$$= -3(x^2) + 4$$

$$= -3x^2 + 4$$

Same!

EVEN
 $f(x) = f(-x)$

12. Is the function even, odd, or neither? Show algebraically.

$$f(x) = 2x^3 - 4x$$

$$f(-x) = 2(-x)^3 - 4(-x)$$

$$= 2(-x^3) + 4x$$

$$= -2x^3 + 4x$$

All signs changed!

ODD
 $-f(x) = f(-x)$

13. Is the function even, odd, or neither? Show algebraically.

$$f(x) = 2x^3 - 4x^2 + 3x - 10$$

$$f(-x) = 2(-x)^3 - 4(-x)^2 + 3(-x) - 10$$

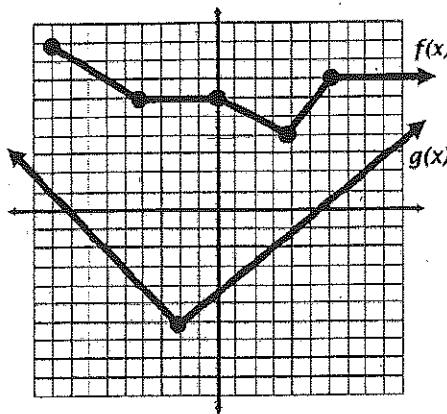
$$= 2(-x^3) - 4(x^2) - 3x - 10$$

$$= -2x^3 - 4x^2 - 3x - 10$$

NO pattern!

Neither.

14. Consider $f(x)$ and $g(x)$ pictured below.



a. Is $f(x)$ a function, relation, or both? Explain.
 Both. Every function is a relation, but all relations are not functions.

b. Is $g(x)$ a function, relation, or both? Explain.
 Both.

c. State the domain & range of $f(x)$ and $g(x)$, using both interval and inequality notation.

$f(x)$:

domain $[-9, \infty); x \geq -9$

range: $\{4, 9\}; 4 \leq y \leq 9$

$g(x)$:

domain $(-\infty, \infty)$

range: $(-6, \infty); y \geq -6$

d. What is $f(2) + g(-2)$?

$$5 + -6$$

$$5 - 6$$

$$\boxed{-1}$$

e. What is $f(g(-8))$?

$$f(0)$$

$$\boxed{6}$$

f. $f(x) = 4$, what is x ?

$$\boxed{x = 4}$$

g. What is $(f \circ g)(8)$?

$$f(8) \cdot g(8)$$

$$(7) \cdot (2)$$

$$\boxed{14}$$

h. What is $f(12) - g(4)$

$$7 - (-1)$$

$$7 + 1$$

$$\boxed{8}$$

i. What is $g(-7) - f(0) + g(-3)$?

$$-1 - 6 + (-5)$$

$$-1 - 6 - 5$$

$$-7 - 5$$

$$\boxed{-12}$$

j. What is $2f(0)$?

$$2(6)$$

$$\boxed{12}$$

k. What is $\frac{2f(2)}{g(4)f(0)}$?

$$\square \quad \frac{2(5)}{(-1)(6)} = \frac{10}{-6} = \boxed{\frac{5}{3}}$$

Midterm Review: Function Transformations

1. Function Notation: $A \cdot f(Bx + C) + D$

What does A, B, C, and D do to a function?

A

- * if negative, the function reflects over x-axis

- * $|A| < 1$ widens/shrinks

- * $|A| > 1$ narrows/stretches

B

- * If negative, the function reflects over the y-axis

C

- * negative means it shifts right

- * positive means it shifts left

D

- * negative means shifts down

- * positive means shifts up.

2. (a) Describe the transformation that occurred? $f(x) = -2(x - 4)^2 + 3$
- * reflected over x-axis
 - * shifted up 3 units
 - * stretched by a factor of 2
 - * shifted right 4 units
- (b) Describe the transformation that occurred? $f(x) = \frac{1}{4}(x + 1)^3 - 2$
- * shrunk by a factor of $\frac{1}{4}$
 - * shifted left 1 unit
 - * shifted down 2 units
- (c) Describe the transformation that occurred? $f(x) = -\sqrt{-x + 6}$
- * reflected over x-axis
 - * reflected over y-axis
 - * shifted left 6 units

3.

a. $f(x) = -(x - 4)^2 - 3$

b. $g(x) = (x + 2)^3 + 2$

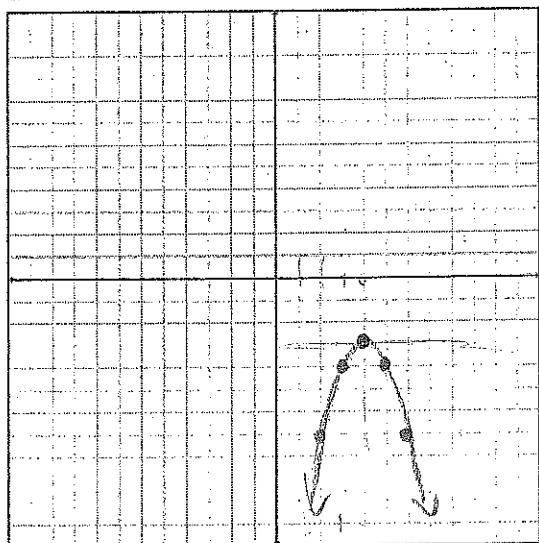
Description of transformation:

* reflected over x-axis

* shifted right 4 units

* shifted down 3 units

Graph:



Domain:

$$-\infty < x < \infty$$

Range:

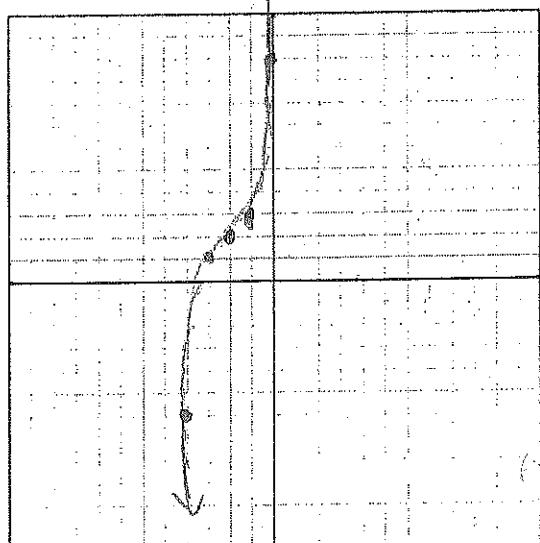
$$y \leq -3$$

Description of transformation:

* shifted left 2 units

* shifted up 2 units

Graph:



Domain:

$$-\infty < x < \infty$$

Range:

$$-\infty < y < \infty$$

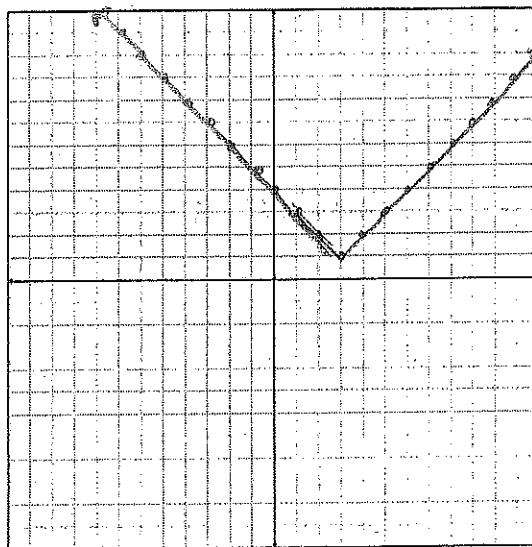
$$c. \quad k(x) = |x - 3| + 1$$

$$d. \quad j(x) = -\sqrt{-x + 2}$$

Description of transformation:

- * shifted right 3 units
- * shifted up 1 unit

Graph:



Domain:

$$-60 < x < 60$$

Range:

$$y \geq 1$$

Domain:

$$x \leq 2$$

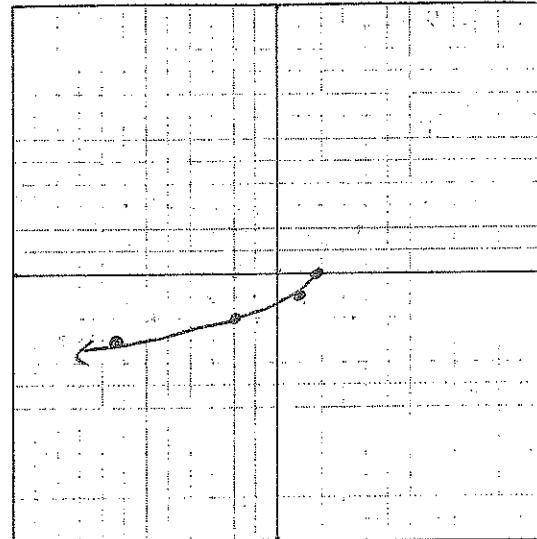
Range:

$$y \leq 0$$

Description of transformation:

- * reflected over x-axis
- * reflected over y-axis
- * shifted left 2 units

Graph:



4. Write an equation to represent each function.

a. An absolute value function is shrunk by $\frac{1}{2}$, shifted 3 units left, and 4 units up.

$$f(x) = \frac{1}{2} |x + 3| + 4$$

b. A cubic function is reflected over the y-axis, and translated 5 units to the left.

$$f(x) = (-x + 5)^3$$

c. A quadratic function is reflected across the x-axis, stretched by 2, and shifted 6 units up and 11 units to the right.

$$f(x) = -2(x - 11)^2 + 6$$

d. A radical function is reflected across both axes, and translated 9 units down.

$$f(x) = -\sqrt{-x} - 9$$

5. Consider the function $d(x) = 4x^2 + 2x$.

a. Describe the transformation that maps the graph of $d(x)$ to $d(x - 3)$.

The function moves right 3 units

b. Write a function that defines $d(x - 3)$.

$$4(x - 3)^2 + 2(x - 3)$$

$$4(x - 3)(x - 3) + 2(x - 3)$$

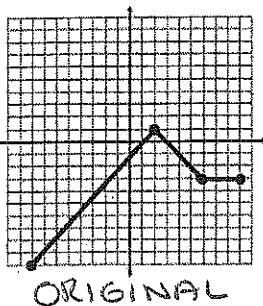
$$4(x^2 - 3x - 3x + 9) + 2x - 6$$

$$4x^2 - 24x + 36 + 2x - 6$$

$$4x^2 - 22x + 30$$

$$4x^2 - 22x + 30$$

6. Consider the function, $j(x)$, pictured below. $a(x) = j(x + 3) - 1$ and $b(x) = j(-x) - 3$.



ORIGINAL

(2, 1)
(9, -3)

(-6, -3)

(-8, -10)

- a. What are the domain and range of $a(x)$?

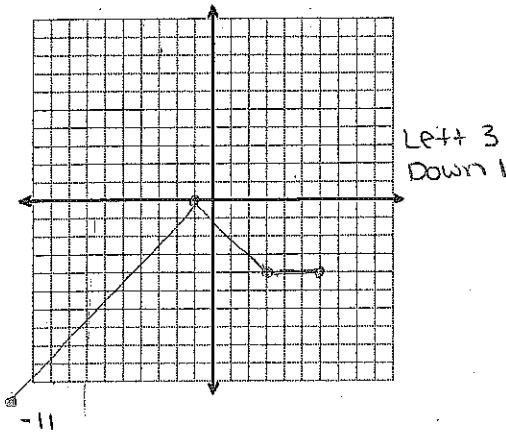
$$D: [-11, 6] \quad R: [-11, 0]$$

Graphs are provided if you wish to use them to help you!

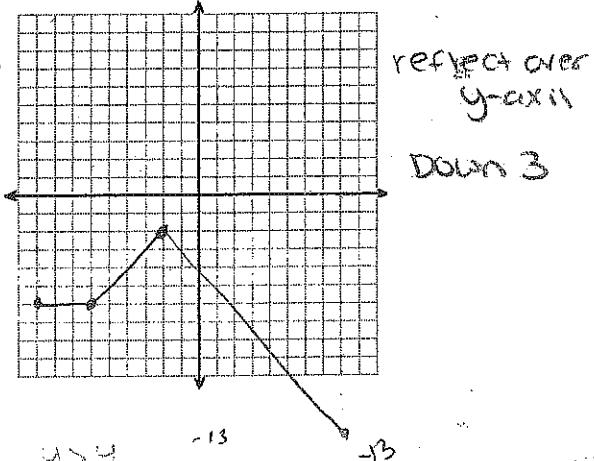
- b. What are the domain and range of $b(x)$?

$$D: [-9, 8] \quad R: [-13, -2]$$

$a(x)$



$b(x)$

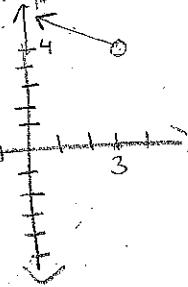


7. The domain of a function, $d(x)$ is $(-\infty, 3)$ and the range is $(4, \infty)$.

Determine the new domain or range for each.

Create an example:

a. $-d(x)$
reflect over X-axis



$$D: (-\infty, 3)$$

$$R: (-\infty, -4)$$

b. $d(-x)$
reflect over y-axis

$$D: (-3, \infty)$$

$$R: (4, \infty)$$

c. $d(x - 7)$
Shift right 7

$$D: (-\infty, 10)$$

$$R: (4, \infty)$$

d. $d(x) + 7$
Shift up 7

$$D: (-\infty, 3)$$

$$R: (11, \infty)$$

8. a. Will the function $f(x) = (x + 1)^2 + 9$ have any real roots? Why or why not?

No real roots. When you solve by using square roots, you must take the square root of -9. That will produce an imaginary answer of $\pm 3i$.

$$\begin{aligned} (x+1)^2 + 9 &= 0 \\ -9 &= -9 \end{aligned}$$

$$(x+1)^2 = -9$$

$$\sqrt{(x+1)^2} = \sqrt{-9}$$

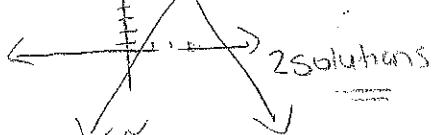
$$\begin{aligned} x+1 &= \pm 3i \\ -1 &= -1 \end{aligned}$$

$$x = \pm 3i - 1$$

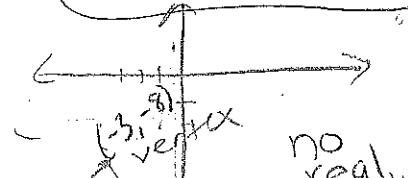
- b. Which equation will not have any real solutions for x , and how do you know?

Equation 1: $-|x - 3| + 8 = 0$

$$(3, 8) = \text{vertex}$$



Equation 2: $-|x + 3| - 8 = 0$



Quick sketches

9. $r(x)$ is a radical function reflected over the x -axis and translated right 2 units.
 $t(x)$ is defined as $-\sqrt{x-2} - 1$, and $s(x)$ is defined as $-\sqrt{x+2}$

a. Compare and contrast the domain and range of $r(x)$ and $t(x)$.

* $r(x)$ and $t(x)$ have the same domain
of $x \geq 2$

* $r(x)$ has a range of $y \leq 0$ and $t(x)$ has a range of $y \leq -1$.

b. Compare and contrast the domain and range of $r(x)$ and $s(x)$.

* $r(x)$ has a domain of $x \geq 2$ and $s(x)$ has a domain of $x \geq -2$.

* $r(x)$ and $s(x)$ have the same range of $y \leq 0$.

c. Compare and contrast the domain and range of $s(x)$ and $t(x)$.

* $s(x)$ has a domain of $x \geq -2$ and $t(x)$ has a domain of $x \geq 2$

* $s(x)$ has a range of $y \leq 0$ and $t(x)$ has a range of $y \leq -1$

10. Consider the functions $b(x) = 5x^4 + 3x^2 + 1$, $c(x) = 2x^2 - x^4$, and $d(x) = x^3 - x$.

a. Prove algebraically whether $d(x)$ is even, odd, or neither.

* $d(x) = x^3 - x$

All signs
changed

ODD

* $d(-x) = (-x)^3 - (-x)$

$-x^3 + x$

b. Prove algebraically whether $(b - c)(x)$ is even, odd, or neither.

$b(x) - c(x)$

$$(5x^4 + 3x^2 + 1) - (2x^2 - x^4)$$

$$5x^4 + 3x^2 + 1 - 2x^2 + x^4$$

$(b-c)(x) = 6x^4 + x^2 + 1$

A

$(b-c)(-x) = 6(-x)^4 + (-x)^2 + 1$

Same signs

EVEN

c. Prove algebraically whether $(b + d)(x)$ is even, odd, or neither.

$b(x) + d(x)$

$(5x^4 + 3x^2 + 1) + (x^3 - x)$

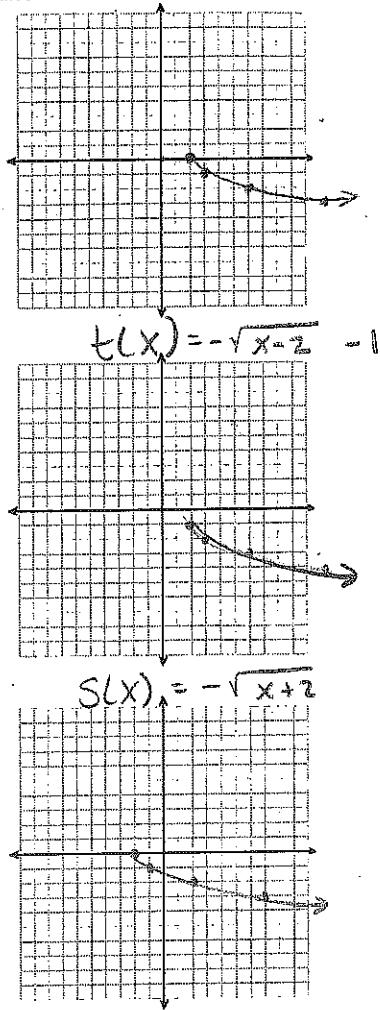
$(b+d)(x) = 5x^4 + x^3 + 3x^2 - x + 1$

NO pattern

$(b+d)(-x) = 5(-x)^4 + (-x)^3 + 3(-x)^2 - (-x) + 1$

$$5x^4 - x^3 + 3x^2 + x + 1$$

Neither



Midterm Review: Algebraic Systems

1. An exam worth 145 points contains 50 questions. Some of the questions are worth two points each, and the rest are worth five points each. How many two-point questions are on the test? How many five-point questions are on the test?

(a) Define the variables $x = \# \text{ 2 point questions}$ $y = \# \text{ 5 point questions}$

(b) Write a system of equations that represents the situation and solve.

$$\begin{aligned} 2x + 5y &= 145 \\ x + y &= 50 \end{aligned} \Rightarrow \begin{aligned} 2x + 5y &= 145 \\ -2x - 2y &= -100 \end{aligned}$$

$$\begin{array}{r} 3y = 45 \\ \hline 3 \\ \boxed{y = 15} \end{array}$$

There were 15 5 point questions on the exam.

2. Solve the system below.

$$\begin{array}{l} r + 4s - t = 9 \quad (1) \\ 6r + 3s - 5t = -11 \quad (2) \\ 4r - 4s - 3t = -26 \quad (3) \end{array} \quad \left\{ \begin{array}{l} r, s, t \end{array} \right\} \quad \boxed{\begin{array}{l} r = -5 \\ s = 3 \\ t = -23 \end{array}}$$

Step 1: Eliminate (1) and (3) for s

$$\begin{array}{r} r + 4s - t = 9 \\ + 4r - 4s - 3t = -26 \\ \hline 5r - 4t = -17 \end{array}$$

Step 4: Substitute $t = -2$ into answer from Step 1.

$$5r - 4(-2) = -17$$

$$\begin{array}{r} 5r + 8 = 17 \\ -8 \quad -8 \\ \hline 5r = 9 \end{array}$$

$$\begin{array}{r} 5r = 9 \\ \hline 5 \\ \boxed{r = -5} \end{array}$$

Step 2: Eliminate (2) and (3) for s

$$\begin{array}{r} 4(6r + 3s - 5t = -11) \\ 3(4r - 4s - 3t = -26) \end{array} \Rightarrow \begin{array}{r} 24r + 12s - 20t = -44 \\ 12r - 12s - 9t = -78 \\ \hline 36r - 29t = -122 \end{array}$$

Step 5: Substitute $t = -2$ and $r = -5$ into original problem

$$r + 4s - t = 9$$

$$-5 + 4s - (-2) = 9$$

$$-5 + 4s + 2 = 9$$

$$-3 + 4s = 9$$

$$+3 \quad +3$$

$$\begin{array}{r} 4s = 12 \\ \hline 4 \\ \boxed{s = 3} \end{array}$$

Step 3: Eliminate answers from Step 1 and 2 for r

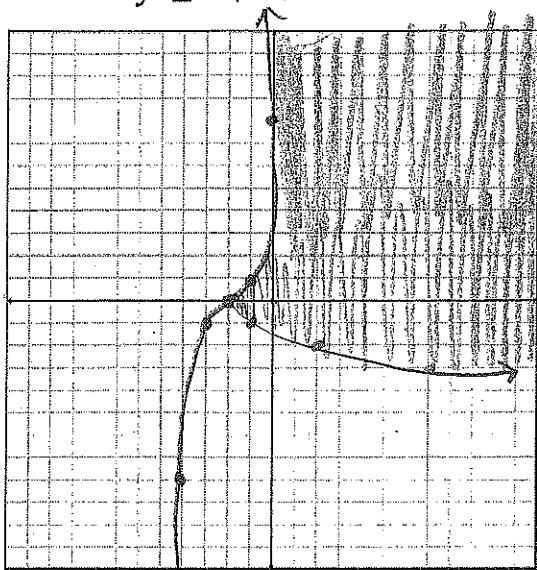
$$\begin{array}{r} -36(5r - 4t = -17) \\ 5(36r - 29t = -122) \end{array} \Rightarrow \begin{array}{r} -180r + 144t = 612 \\ 180r - 145t = -610 \\ \hline -t = 2 \end{array}$$

$$\begin{array}{r} -1 \quad -1 \\ \hline t = -2 \end{array}$$

3. a. Sketch a graph of the system, and shade the solution set.

$$y \leq (x + 2)^3$$

$$y \geq -\sqrt{x + 2}$$



- b. State any point that is included in the solution set.

4. Determine the solution to each system.

a. $5x + 4y = 36$ $4y = -5x + 36$
 $5x + 4y = 12$ $y = -\frac{5}{4}x + 9$

\downarrow

$4y = -5x + 12$
 $y = -\frac{5}{4}x + 3$

b. $y = -4x + 4$
 $y = -\frac{1}{3}x - 7$

1 solution

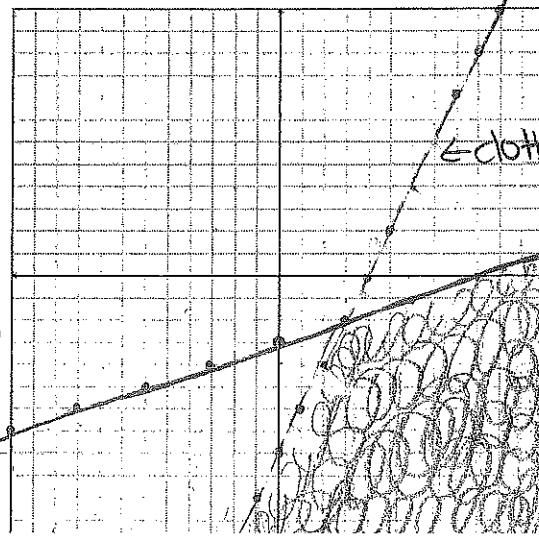
Parallel lines: (No Solutions)

5. Sketch the solution to each system of inequalities.

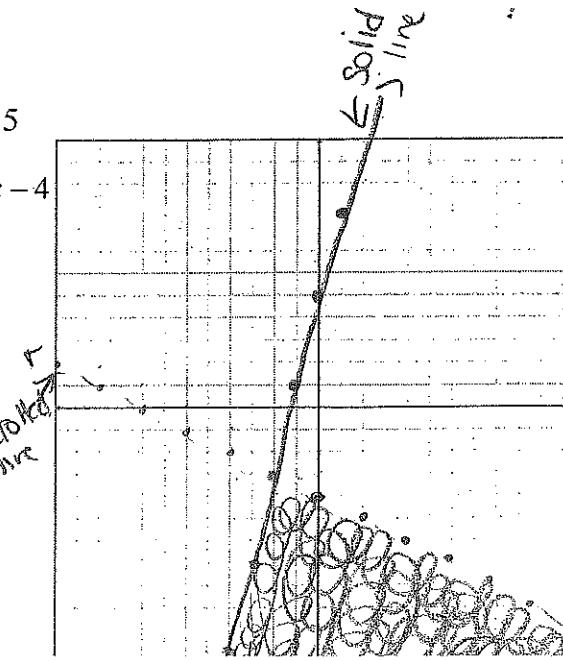
a. $x - 3y \geq 9$
 $2x - y > 8$

$\frac{-3y \geq -x + 9}{-3} \quad \frac{2x - y > 8}{-3}$
Divide by negative
flip sign.
 $y \leq \frac{1}{3}x - 3$

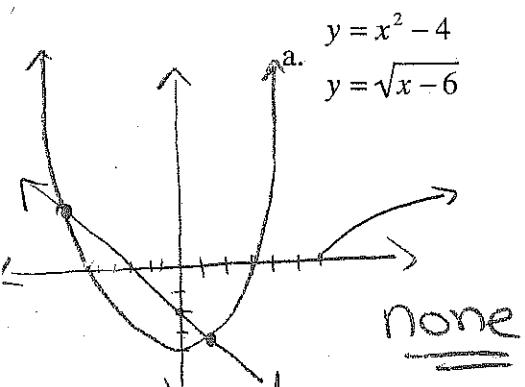
$2x - y > 8$
 $-y > -2x + 8$
Divide by negative
flip sign.
 $y < 2x - 8$



b. $y \leq 4x + 5$
 $y < -\frac{1}{2}x - 4$



6. Determine the number of solutions each system will have.



QUICK sketches

b. $y = x^2 - 4$
 $y = -2x - 2$

2 Solution
~~11~~

7. Is $(1, -2)$ a solution to the system below? Explain.

$$\begin{array}{ll} x \\ y \\ \hline y = (x-1)^2 - 2 & y = (x-1)^2 - 2 \\ y = (1-1)^2 - 2 & -2 = (1-1)^2 - 2 \\ y = -3x + 1 & -2 = (0)^2 - 2 \\ & -2 = -2 \quad \checkmark \end{array}$$

$$\begin{array}{l} y = -3x + 1 \\ -2 = -3(1) + 1 \\ -2 = -3 + 1 \\ -2 = -2 \quad \checkmark \end{array}$$

Both check, so yes it is a solution!!

8. Does every system of non-linear equations have exactly one solution? Explain.

No! Problem 6a is an example of a nonlinear system with no solutions.

9. Cary babysits for \$9 per hour and tutors for \$15 per hour. She cannot work more than 17 hours per week. She can tutor for no more than 4 hours per week, and she wants to make at least \$95. Let x represent the number of hours Cary babysits per week and y represent the number of hours she tutors each week. Write a system of inequalities that could be used to represent the situation.

$x = \# \text{ hours babysitting}$ $y = \# \text{ hours tutoring}$

System of inequalities

$$\left\{ \begin{array}{l} 9x + 15y \geq 95 \\ x + y \leq 17 \\ y \leq 4 \end{array} \right.$$

10. Consider the equation $\frac{-5}{x} = |x^2 - 3| + 7$. Describe the simplest way to determine the solution(s) to the equation. You do not need to actually solve.

* Graph $\frac{-5}{x}$

* Graph $|x^2 - 3| + 7$

* Find the point(s) of intersection, which would be the solution(s).

11. How could you solve the equation $5x^2 - 8x = 19$ without using the quadratic formula?

* Subtract 19 from both sides.

* Graph $5x^2 - 8x - 19$ on calculator.

* Look for the x-intercepts.

12. The manufacturing cost of producing x units of a product is modeled by the function $c(x) = 12x$, and the revenue earned from selling x units of the product is modeled by the function $r(x) = x^2 - 6x$. What is the minimum number of units the company should create and sell in order to break even or make a profit?

$$c(x) = r(x)$$

Set equal

$$12x = x^2 - 6x \quad \text{solve by zero product property.}$$

$$\frac{12x}{12x} - 12x$$

$$0 = x^2 - 18x$$

$$0 = x(x - 18)$$

$$x = 0 \quad x - 18 = 0$$

$$(0,0) \quad (18,0)$$

They will break even after selling 18 units of the product.

13. Consider the function $f(x) = |x + 4| - 3$.

a. Create any function $j(x)$, for which there will be exactly one solution to $j(x) = f(x)$, and explain the reasoning that supports your choice.

$$j(x) = -|x + 4| - 3$$

b. Create any function $p(x)$, for which there will be no real solutions to $f(x) = p(x)$, and explain the reasoning that supports your choice.

$$p(x) = -|x + 4| - 6$$

c. Create any function $m(x)$, for which there will be infinitely many solutions to $m(x) = f(x)$, and explain the reasoning that supports your choice.

$$m(x) = |x + 4| - 3$$

Same equations overlap, which means they have infinite many solutions.

Midterm Review: Quadratics

1. Solve each equation using square roots.

a. $(r-9)^2 - 3 = -2$

$$\begin{array}{r} +3 \\ \hline (r-9)^2 = 1 \end{array}$$

$$\sqrt{(r-9)^2} = \sqrt{1}$$

$$r-9 = \pm 1$$

$$\begin{array}{l} r-9 = 1 \quad \leftarrow \\ \hline r-9 = 1 \end{array} \quad \begin{array}{l} r-9 = -1 \\ \hline r-9 = -1 \end{array}$$

$$\boxed{r=10} \quad \boxed{r=8}$$

b. $5p^2 + 245 = 0$

$$\begin{array}{r} -245 \\ \hline 5p^2 = -245 \\ \hline 5 \end{array}$$

$$p^2 = -49$$

$$\sqrt{p^2} = \sqrt{-49}$$

no real solution
→ NO!!

2. Solve the equation using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Round your answer to the hundredth.

$$3x^2 = 16 + 2x \quad 3x^2 - 2x - 16 = 0$$

$$\begin{array}{l} a = 3 \\ b = -2 \\ c = -16 \end{array}$$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-16)}}{2(3)}$$

$$\frac{2 \pm \sqrt{196}}{6}$$

$$\frac{2 \pm 14}{6}$$

$$\begin{array}{l} \frac{2+14}{6} \\ \hline \frac{20}{6} \\ \boxed{3.33} \end{array} \quad \begin{array}{l} \frac{2-14}{6} \\ \hline \frac{-12}{6} \\ \boxed{-2} \end{array}$$

3. Factor each expression completely.

a. $(18m^3 - 27m^2) + (4m - 6)$

$$9m^2(2m-3) + 2(2m-3)$$

$$\boxed{(2m-3)(9m^2+2)}$$

b. $b^4 - 81$

$$(b^2 - 9)(b^2 + 9)$$

$$\boxed{(b-3)(b+3)(b^2+9)}$$

c. $r^3 - 5r^2 - 24r$

$$r(r^2 - 5r - 24)$$

$$\boxed{r(r-8)(r+3)}$$

d. $49k^2 - 169$

$$\boxed{(7k-13)(7k+13)}$$

e. $3x^2 - 51x + 216$

$$\boxed{3(x^2 - 17x + 72)}$$

$$\boxed{3(x-9)(x-8)}$$

f. $5q^2 + 13q - 6$

$$\boxed{(5q-2)(q+3)}$$

g. $2x^4 - 9x^2 + 4$

$$\begin{array}{r} (2x^2 - 1)(x^2 - 4) \\ \hline (2x^2 - 1)(x-2)(x+2) \end{array}$$

h. $(3x^3 + x^2)(12x - 4)$

$$x^2(3x+1) - 4(3x+1)$$

$$(x^2 - 4)(3x+1)$$

$$\boxed{(x-2)(x+2)(3x+1)}$$

4. Expand and write your final answer in standard form.

(a) $(x - 5)^2$

$$(x-5)(x-5)$$

$$x^2 - 5x - 5x + 25$$

$$\boxed{x^2 - 10x + 25}$$

(b) $(2x - 3)(x + 8)$

$$2x^2 + 16x - 3x - 24$$

$$\boxed{2x^2 + 13x - 24}$$

5. Determine the y-intercept, axis of symmetry, and vertex for each function.

a. $f(x) = 3x^2 - 6x + 8$

b. $y = -(x - 1)^2 + 3$

c. $y = x^2 + 8x + 15$

$$-(x-1)(x-1) + 3$$

$$-[x^2 - 2x + 1] + 3$$

$$-x^2 + 2x - 1 + 3$$

$$y = -x^2 + 2x + 2$$

$(0, 0)$ y-intercept: $(0, 8)$

y-intercept: $(0, 2)$

y-intercept: $(0, 15)$

$\frac{-b}{2a}$ Axis of Symmetry: $x = 1$

Axis of Symmetry: $x = 1$

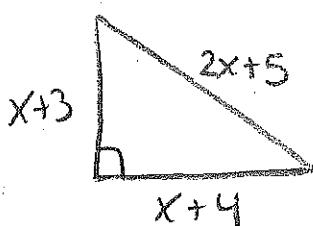
Axis of Symmetry: $x = -4$

$(\frac{-b}{2a}, f(\frac{-b}{2a}))$ Vertex: $(1, 5)$

Vertex: $(1, 3)$

Vertex: $(-4, -1)$

6. The legs of a right triangle have lengths $(x + 4)$ units, and $(x + 3)$ units, and the hypotenuse has a length of $(2x + 5)$ units. Determine the perimeter of the triangle.



$$(x+3)^2 + (x+4)^2 = (2x+5)^2$$

$$(x+3)(x+3) + (x+4)(x+4) = (2x+5)(2x+5)$$

$$x^2 + 6x + 9 + x^2 + 8x + 16 = 4x^2 + 20x + 25$$

$$\cancel{2x^2} + 14x + 25 \neq 4x^2 + 20x + 25$$

$$\cancel{-2x^2} - 4x - 25 = -2x^2 - 14x - 25$$

Perimeter: $3 + 5 + 4$

12 units

$$0 = 2x^2 + 6x$$

$$0 = 2x(x + 3)$$

$$\begin{array}{l} 2x = 0 \\ x = 0 \\ x + 3 = 0 \\ x = -3 \end{array}$$

ignore

7. A ball is thrown into the air from the top of a building, and its altitude in feet is given by the equation $h = -16t^2 + 64t + 48$, where t is the elapsed time in seconds.

a. What is the maximum altitude of the ball?

Find vertex

$$\frac{-b}{2a} \Rightarrow \frac{-64}{2(-16)} \Rightarrow \frac{-64}{-32} \Rightarrow 2$$

$$-16(2)^2 + 64(2) + 48$$

$$-64 + 128 + 48$$

112

t, h
 $(2, 112)$

The max. altitude
is 112 feet

b. How long will it take the ball to hit the ground? Explain.

$$0 = -16t^2 + 64t + 48$$

$$n=0 \quad -64 \pm \sqrt{(64)^2 - 4(-16)(48)}$$

$$2(-16)$$

$$\frac{-64 + \sqrt{17104}}{-32} (-.65)$$

$$\frac{-64 - \sqrt{17104}}{-32}$$

About
4.65
seconds.

↑
Cannot be factored.

use quadratic formula.

8. An object is launched, and its altitude, h , in feet after t seconds is given by the equation

$$h = -4.9t^2 + 19.6t + 58.8.$$

- a. Does the object start on the ground? How do you know?

No! y-intercept at $(0, 58.8)$

Starts 58.8 feet off the ground.

- b. When will the object reach its maximum height, and what is that height?

$$\frac{-b}{2a} = \frac{-19.6}{2(-4.9)} = \frac{-19.6}{-9.8} = 2 \quad \text{vertex}$$

$$f\left(\frac{-b}{2a}\right) = -4.9(2)^2 + 19.6(2) + 58.8 = 78.4$$

$$\text{vertex} = (2, 78.4)$$

- c. When will the object hit the ground? Explain.

$$0 = -4.9t^2 + 19.6t + 58.8 \quad | \quad h=0$$

$$0 = -4.9(t^2 - 4t - 12)$$

$$0 = -4.9(t - 6)(t + 2)$$

$$\begin{aligned} t+6 &= 0 & t+2 &= 0 \\ t &= 6 & t &= -2 \\ &\uparrow && \uparrow \\ &\text{ignore negative time} && \end{aligned}$$

The object will hit the ground after 6 seconds.

9. A rectangular picture frame is three inches shorter than it is wide. The frame has an area of 108 square inches. Determine the perimeter of the frame.

$$\begin{array}{l|l} \text{Length: } (x-3) & 9 \\ \text{Width: } x & 12 \end{array}$$

$$\begin{array}{l} \text{perimeter} = 2L + 2W \\ 2(x-3) + 2(12) \\ 18 + 24 \\ 42 \text{ inches} \end{array}$$

10. Sketch a graph of each parabola.

a. $f(x) = x^2 + 4x + 4$

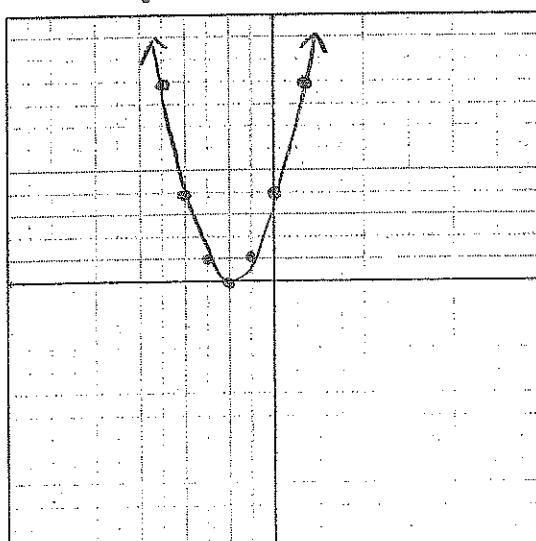
axis of sym: $x = -2$

vertex: $(-2, 0)$

y-int: $(0, 4)$

other points:

$$\begin{array}{ll} (-1, 1) & (-3, 1) \\ (1, 9) & (5, 9) \end{array}$$



Area = length * width

$$108 = (x-3)(x)$$

$$108 = x^2 - 3x - 108$$

$$0 = x^2 - 3x - 108$$

$$0 = (x + 12)(x - 9)$$

$$\begin{array}{l} x+12=0 \\ x=-12 \\ x=12 \end{array}$$

$$\begin{array}{l} x+9=0 \\ x=-9 \\ x=9 \end{array}$$

IGNORE

b. $f(x) = x^2 + 2x - 3$

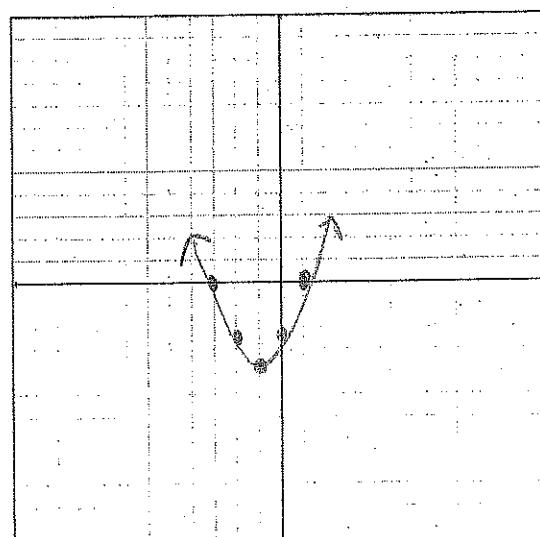
axis of sym: $x = -1$

vertex: $(-1, -4)$

y-intercept: $(0, -3)$

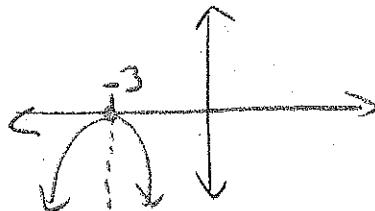
x-intercept: $(x+3)(x-1) = 0$

$$(-3, 0)(1, 0)$$



11. The parabolas in this problem are all the same width as the parent function, $y = x^2$.

Part A: A parabola's vertex is the same as its x-intercept, and its axis of symmetry is the line $x = -3$. The parabola opens downwards. What is the equation of the parabola?

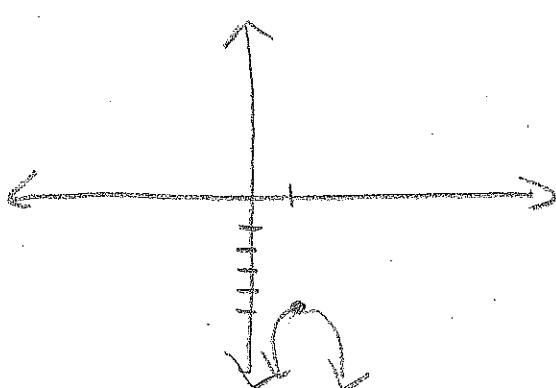


$$y = -(x+3)^2$$

(-3, 0)

Part B: A quadratic function has no real solutions, and its vertex is at the point (1, -5).

What is the equation for this function?



$$y = (x-1)^2 - 5$$

12. Simplify. Keep in radical form.

(a) $\sqrt{27}$

$$\frac{\sqrt{9} \cdot \sqrt{3}}{3\sqrt{3}}$$

(b) $\sqrt{40}$

$$\frac{\sqrt{4} \cdot \sqrt{10}}{2\sqrt{10}}$$

(c) $\sqrt{300}$

$$\frac{\sqrt{100} \cdot \sqrt{3}}{10\sqrt{3}}$$